Development of Consistent Geologic Material Failure

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Name of Event

Name of presenter

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Acknowledgements

- **Sandia National Laboratories**
  - Arlo Fossum (original model developer -- now at BP)
- **University of Utah**
  - Rebecca Brannon (model developer) and students
- **Army Research Laboratory**
  - Brain Leavy, Josh Houskamp, Melissa Love, Clint Nicely

Overview

- **Kayenta Material Model**
  - Pressure-dependent yield stress
  - Softening response
  - Spatial variability
- **Failure Response Modeling**
  - Brittle material response
  - Dilation of failed material under shear
- **Calibration Approaches for Kayenta Model**
- **Conclusions**

Kayenta Development

- A. Fossum began development of geologic material model (1995) with many of these attributes.
- Subsequent development was pursued by R.M. Brannon and O.E. Strack which resulted in the Kayenta material model.
- Currently under joint development with University of Utah
- Active implementation and use in selected Sandia finite element and finite volume codes

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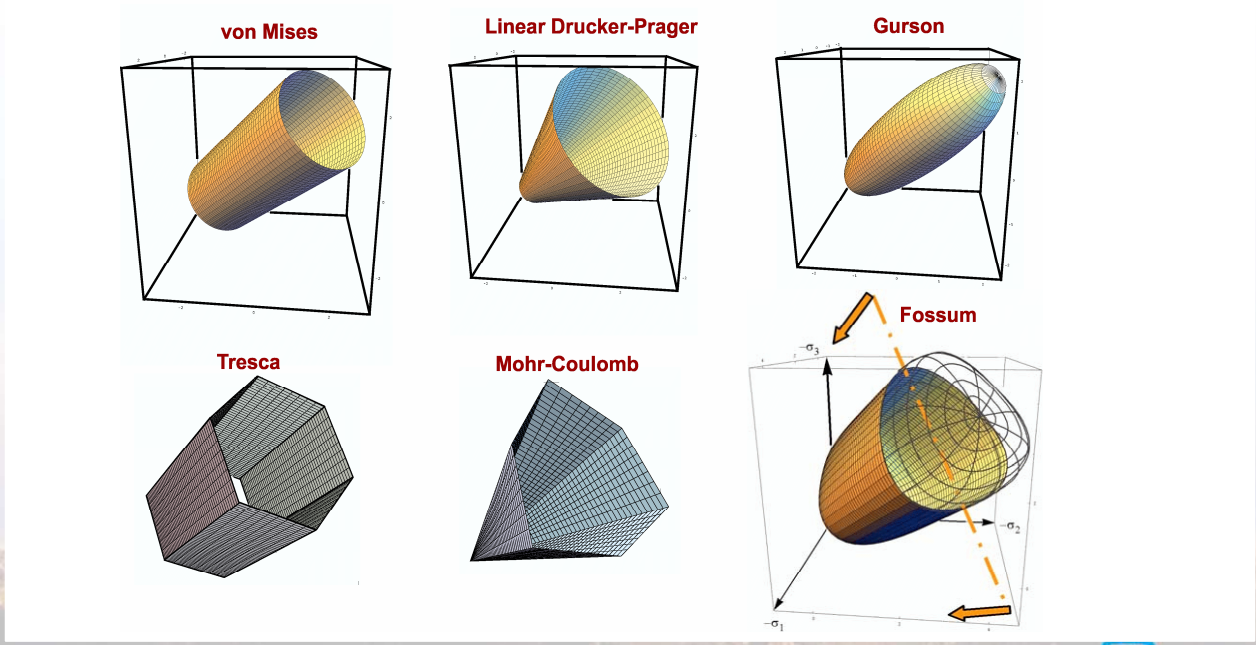
Kayenta Material Model

Key features

- 3 invariant, mixed hardening, continuous surface cap plasticity
- Pressure and shear dependent compaction of pores
- Strain-rate independent or strain-rate sensitive yield surface
- Nonlinear kinematic hardening accounting for Baushinger effect
- Varying TXE/TXC strength ratio through third invariant dependence
- Peak shear threshold marking onset of softening and for fully damage material



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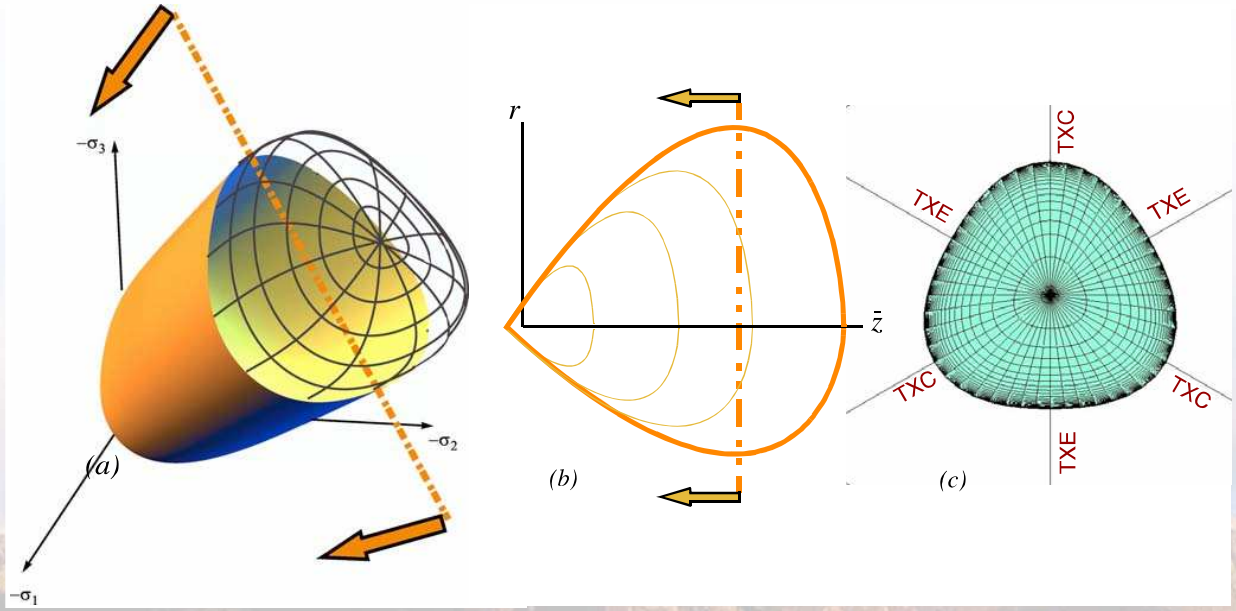
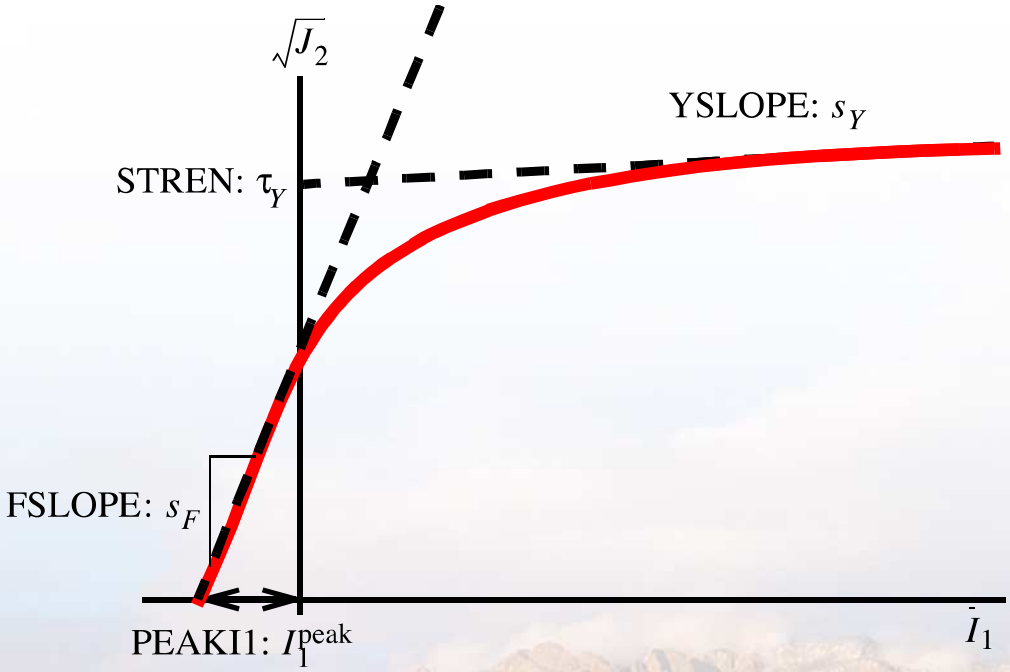


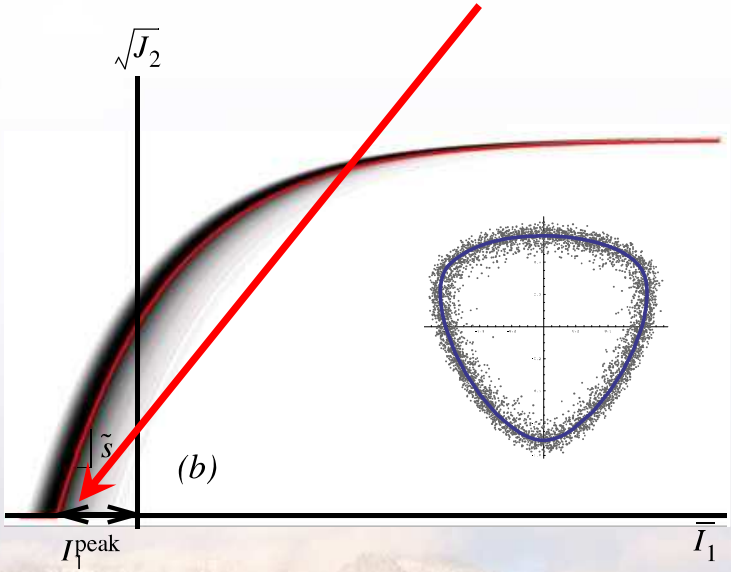
Kayenta Material Model

Parameterization of yield surface through yield function variables

- I_1^{Peak}, S_F, τ_Y, S_Y



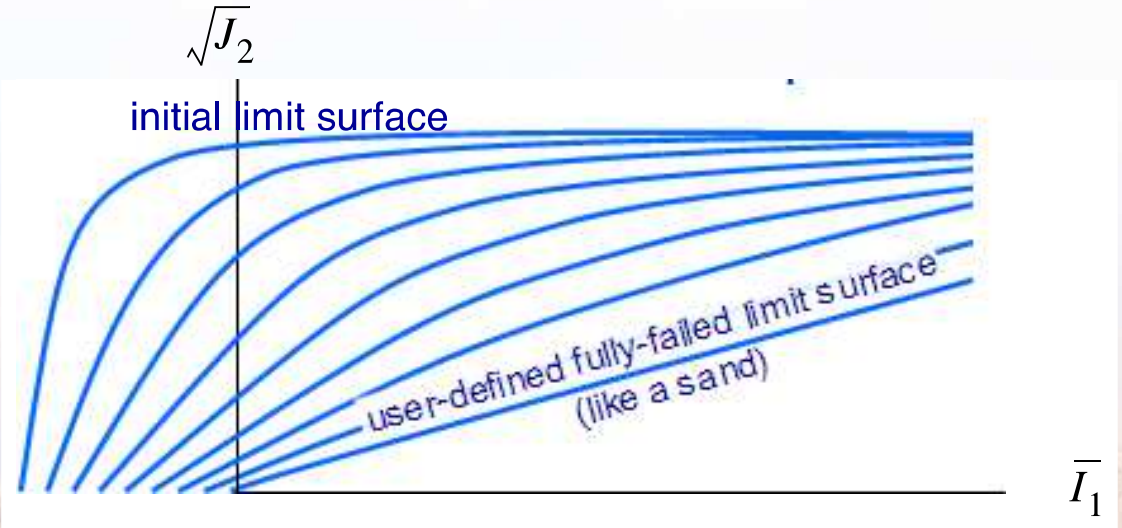
Statistically perturbed I_1^{Peak}



Kayenta Material Model

Softening response

$$I_1^{\text{Peak}} = (1-D)I_{10}^{\text{Peak}} + DI_{1F}^{\text{Peak}}$$
$$S_F = (1-D)S_{F0} + DS_{FF}$$
$$\tau_Y = (1-D)\tau_{Y0} + D\tau_{YF}$$
$$S_Y = (1-D)S_{Y0} + DS_{YF}$$



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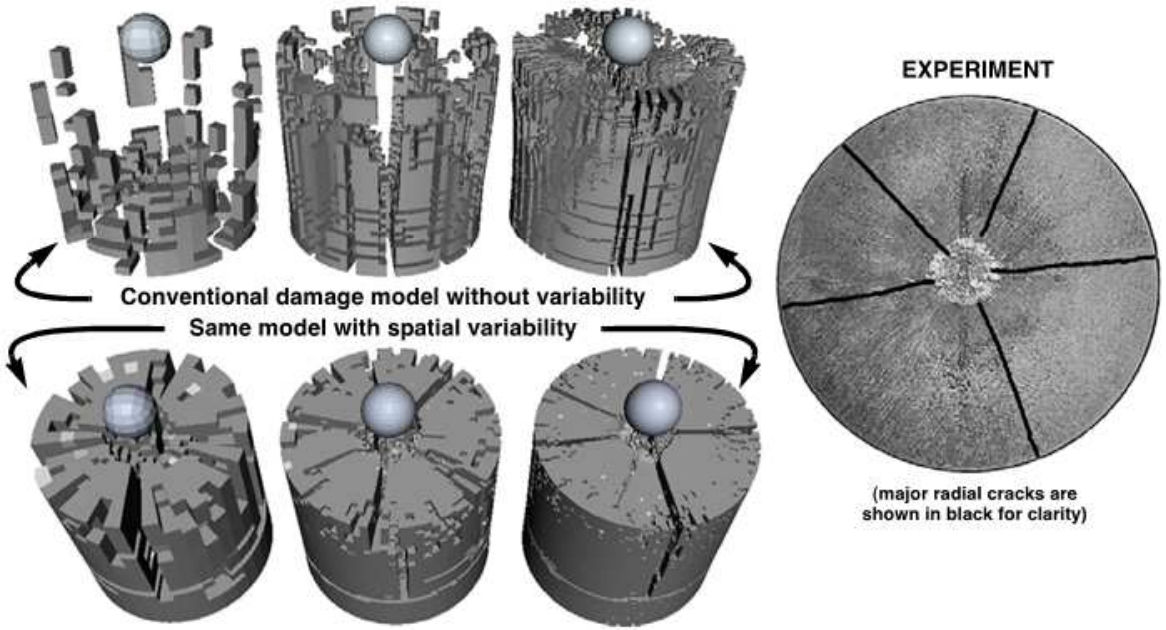
Kayenta Material Model

Sphere indentation with softening

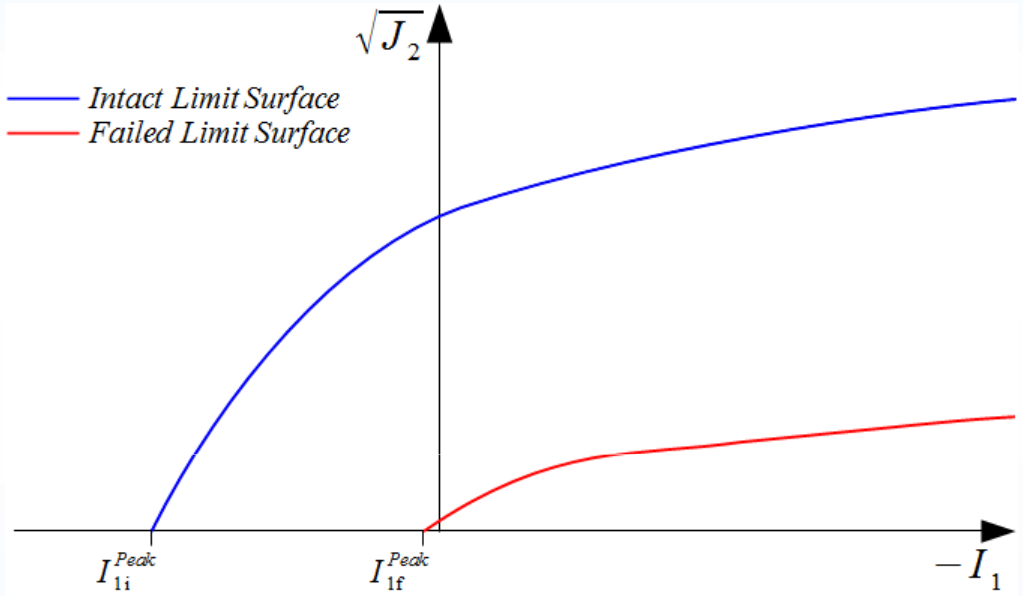


Kayenta with EOS and Void Insertion Models

- Thermodynamic quantities needed by Kayenta are bulk and shear moduli.
- For low strain rates, these quantities are sufficient. However, at higher strain rates, compressions or temperatures a mechanical equation of state is useful for computing material response.
- Other thermodynamic quantities needed to complete a finite element cycle: pressure, sound speed and temperature.
- Thus, Kayenta should be paired with an equation of state model (i.e, Mie-Gruneisen or tabular EOS) to provide the material description required by the code.
- Condensed materials can only sustain a certain amount of tension before breaking. We seek to limit unphyiscal tension through a void insertion model
- When the pressure of a material falls below a tensile limit, the density is increased, raising the pressure above the limiting tension.
- To conserve mass, the volume of the material decreases. The difference between the original volume and reduced volume is the volume of “void” inserted into the element.

Kayenta with Void Insertion Model

- The void insertion model tensile limit, corresponds to I_1^{Peak}
- As the material fails, the tensile limit for void insertion reduces.
- Void insertion model with Kayenta seeks to avoid stress states where the means stress is below $Peak\ I_1$
- In mixed material cells (i.e., gas and solid), the Kayenta material can often be subjected to excessive expansion. Void insertion keeps I_1 from becoming too tensile and failing Kayenta.



$$I_1^{Peak} = (1-D)I_{1o}^{Peak} + DI_{1f}^{Peak}$$

$$D \equiv Damage, \quad vi \equiv \text{after void insertion}$$

$$P(\rho) < -I_1^{Peak}, \quad P_{vi}(\rho_{vi}) = -I_1^{Peak}$$

$$\rho_{vi}\phi_{vi} = \rho\phi \quad \text{(for mass conservation)}$$

$$\phi_{vi} = \frac{\rho}{\rho_{vi}}\phi, \quad \phi_{vi} < \phi \quad \text{(volume fraction after void insertion)}$$

Deformation and Density Consistency

- **Void insertion reduces material volume, raising density (from A to B until the threshold pressure is reached.**
- **Newton iteration to find ρ_A**

$$P(\rho_A) = P_A < P_{frac}$$

$$P(\rho_B) = P_{frac}$$

$$\rho_i = \rho_{i-1} + \frac{P_{frac} - P_{i-1}}{dP/d\rho}$$

- **Deformation rate modification for void insertion.**

$$\mathbf{D} \equiv \text{Deformation rate}$$

$$\Delta\varepsilon_{vol} = tr(\mathbf{D})\Delta t$$

- **Because density modified by void insertion, need to compute consistent volume change (trace of deformation rate).**

$$\Delta\varepsilon_{vol} = \varepsilon_{volB} - \varepsilon_{volA} = \ln\left(\frac{\rho_o}{\rho_B}\right) - \ln\left(\frac{\rho_o}{\rho_A}\right) = \ln\left(\frac{\rho_A}{\rho_B}\right)$$

$$tr(\tilde{\mathbf{D}}) = \frac{\Delta\varepsilon_{vol}}{\Delta t}$$

$$\tilde{\mathbf{D}} = Deviator(\mathbf{D}) + \frac{1}{3}tr(\tilde{\mathbf{D}})\mathbf{I}$$

Dilatation

- **We assume that the equation of state model provides the pressure calculation for the material.**
- **However, for a failed material, Kayenta will calculate a dilated state, modeling expansion or increased pressure due to crushed material motion under shear.**
- **When the material dilates, the pressure form the equation of state is no longer consistent with the mean stress computed by the Kayenta model.**
  - Material density decreasing while mean stress increasing
- **Recognize that dilatancy results from the interaction among failed (crushed) material particles. Volume will expand, implying formation of voids in the material matrix.**
- **The pressure then would be representative of the density of the solid material (i.e., without the voids caused by the dilation).**

Resolution of Dilation Effects

- To bring the equation of state model into consistency with the dilated state computed by Kayenta, a solid density is computed for the dilated mean stress.
- The solid density will be greater than the material density which includes the voids due to the volumetric expansion of the dilating material.
- Using an iterative scheme similar to void insertion, the equation of state is iterated on density until the dilated pressure is obtained. The resulting density is considered the solid density.

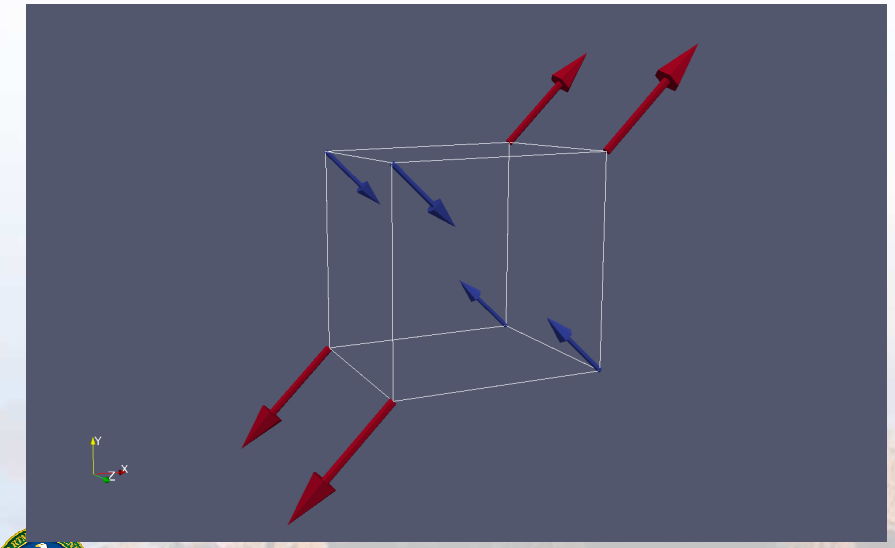
$$\rho_i = \rho_{i-1} + \frac{(-I_1 - P_{i-1})}{dP/d\rho}$$

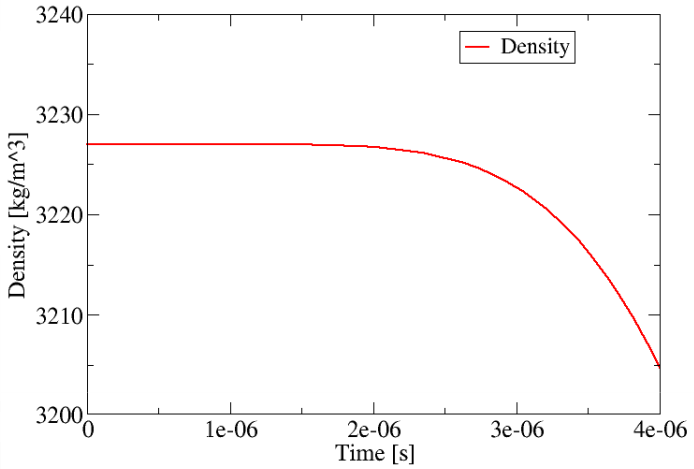
Convergence when $P = -I_1$

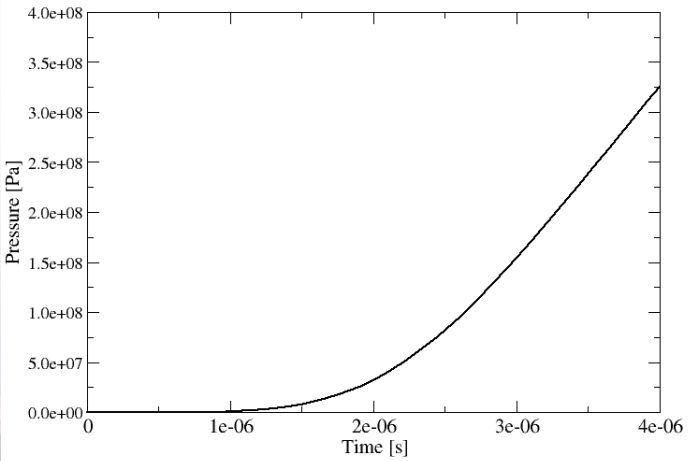
Solid density : $\rho_s = \rho_i$

Demonstration of Dilatation of Failed Material in Kayenta

- Single element subjected to shear forces with gradually increasing magnitude.
- Pressure increases, density decreases, indicating dilatation.
- Coherence (1-Damage), decreases.
- Solid density diverges from density.







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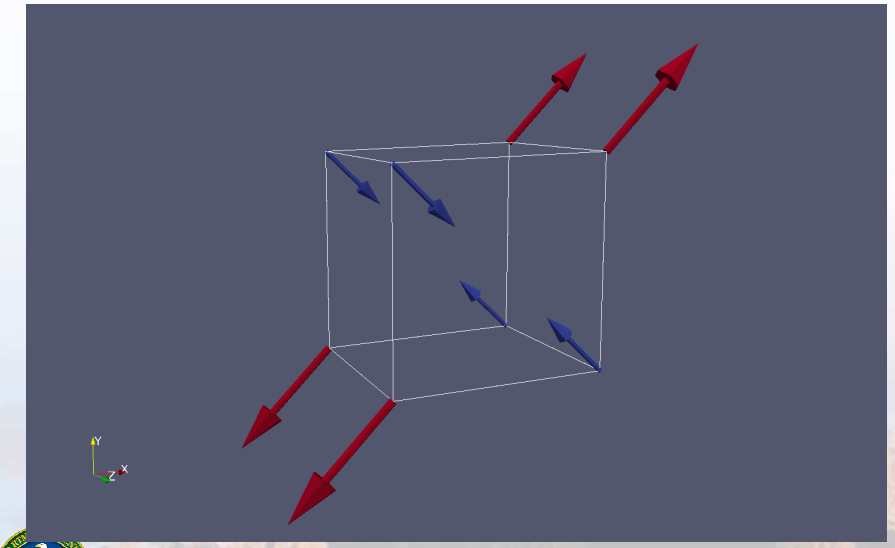

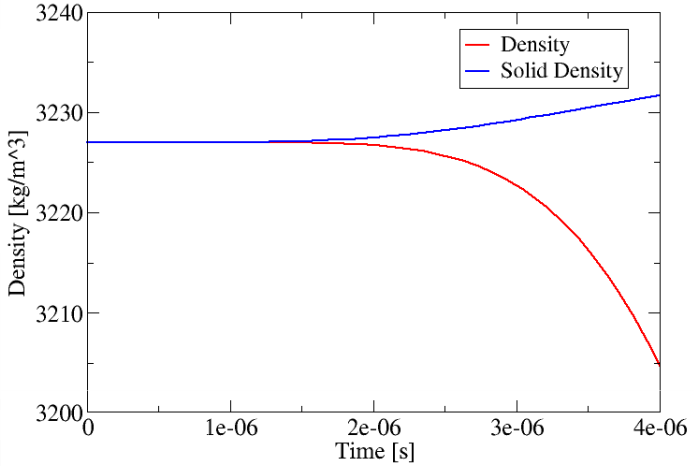

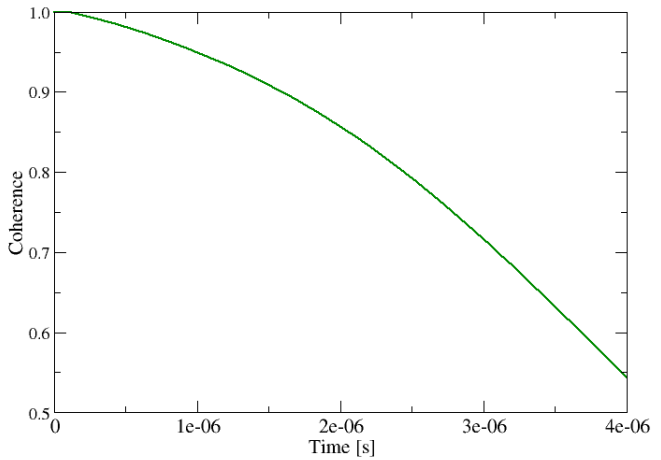

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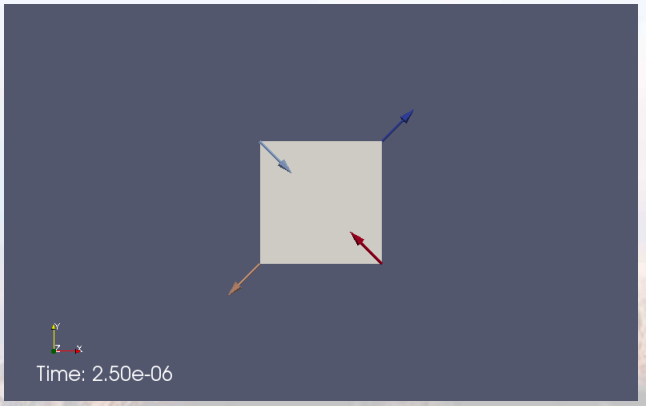
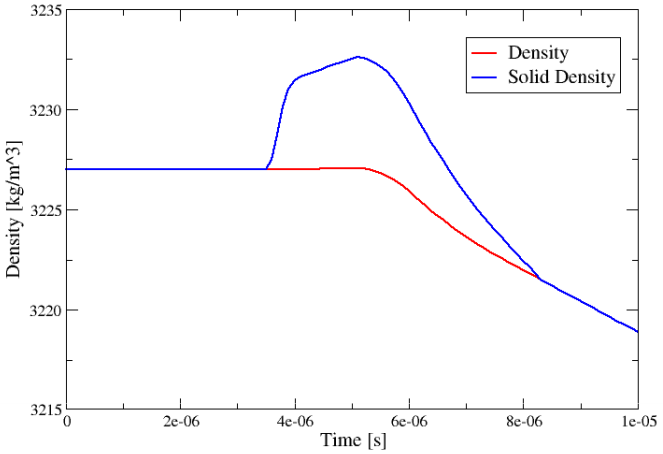
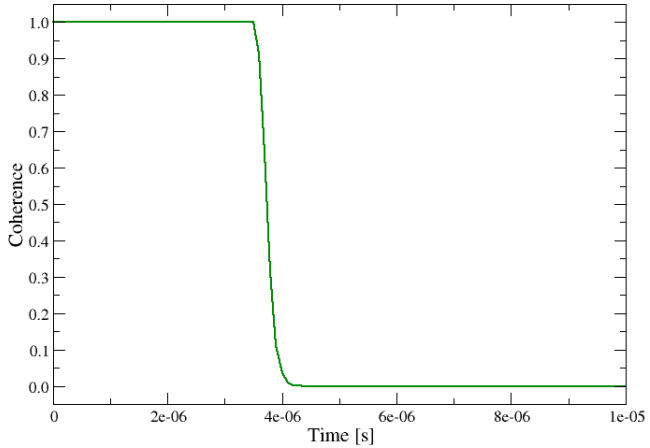
Unloading of Failed Material from Dilated State

- Single element loaded in shear (up to 5 μs), then extended vertically through 8 μs.
- Material dilates upon failure (coherence drops).
- Unloading causes solid density to reduce until it reaches material density.







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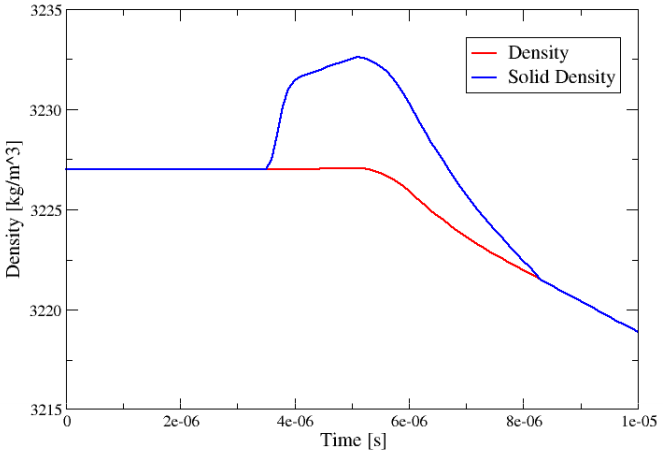
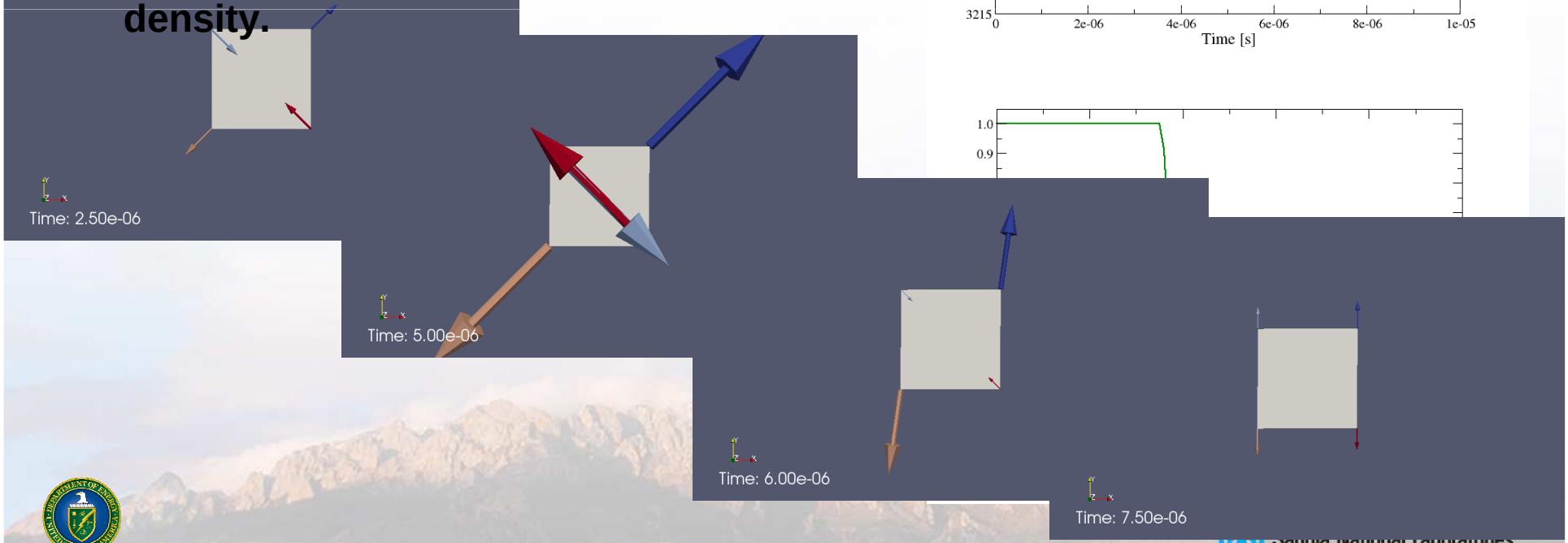
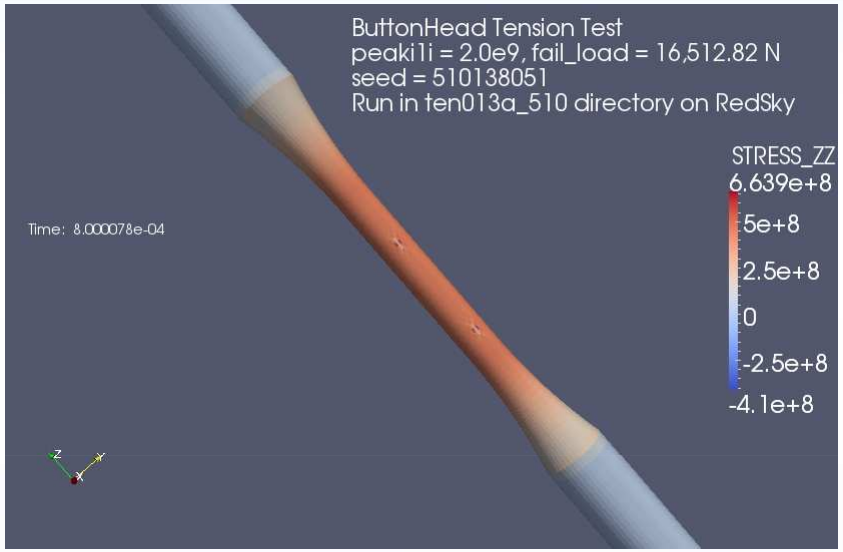
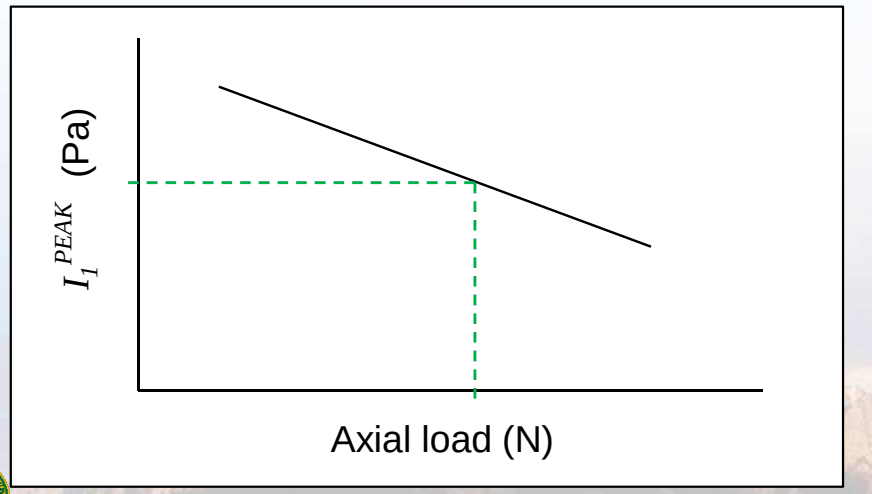


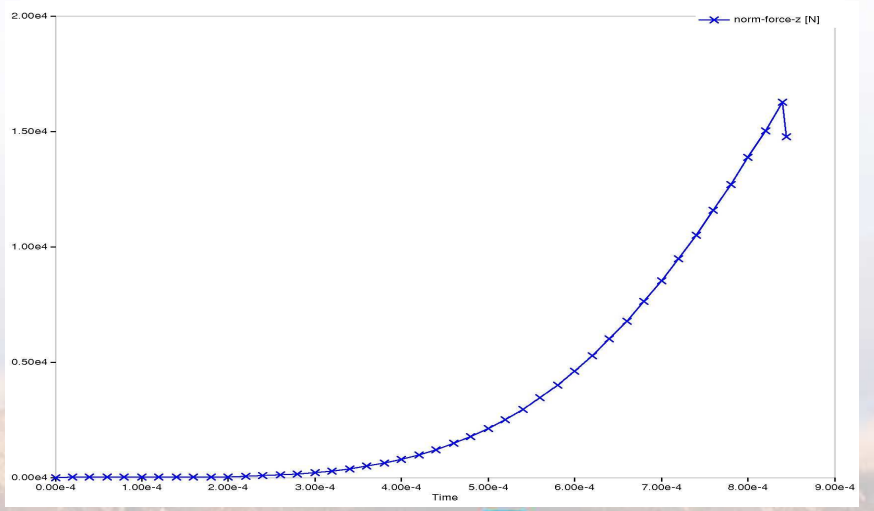


Demonstration of Tensile Failure of Brittle Material with Kayenta

- **Button head tension test**
- **Cylindrical test specimen with tapered ends and button head for mounting in testing apparatus.**
- **Test provides maximum tensile load**
- **Series of calculations for a set of spatially variability cases run to tensile failure.**
- **I_1^{PEAK} selected from results of calculations over a series of I_1^{PEAK} trials**







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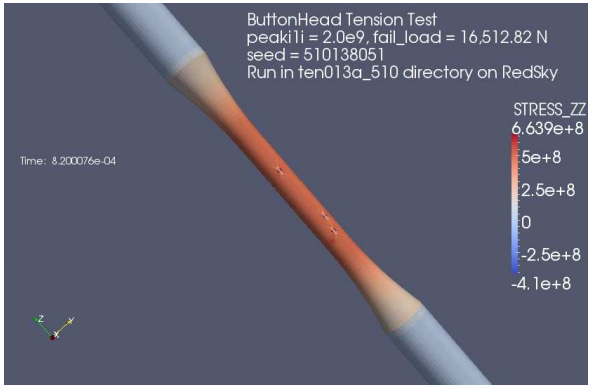

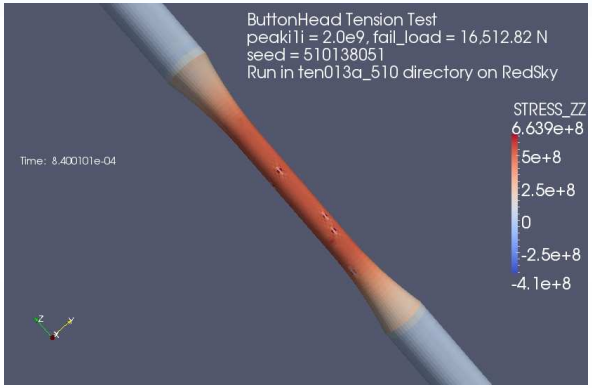

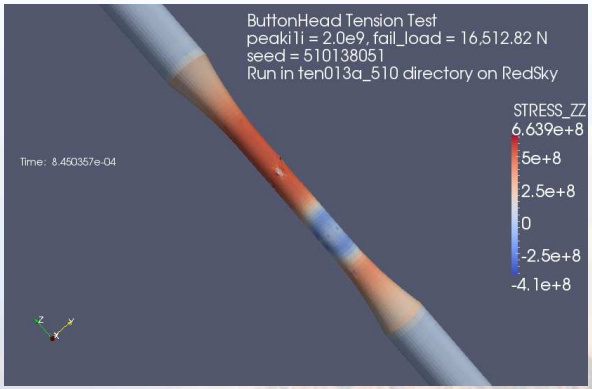

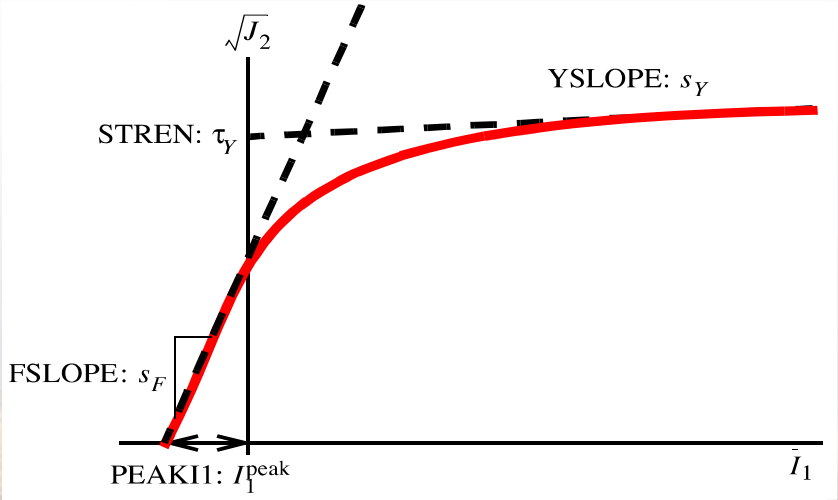

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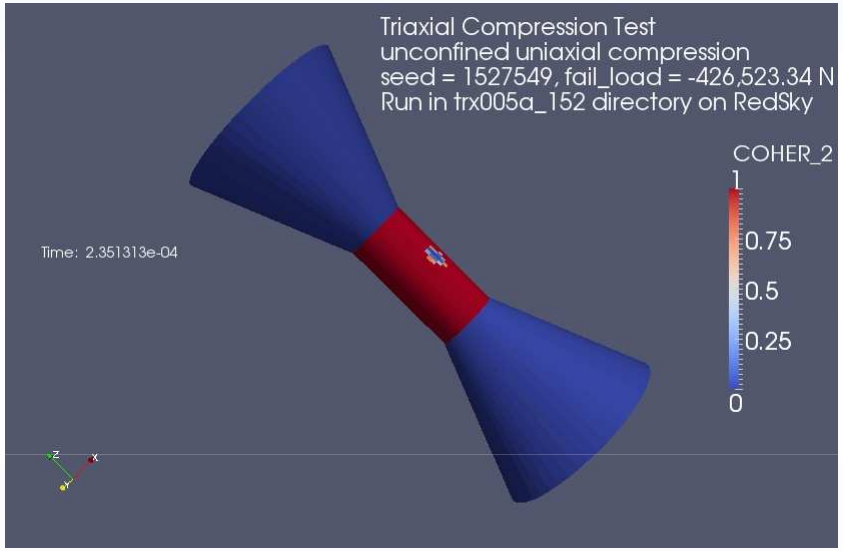
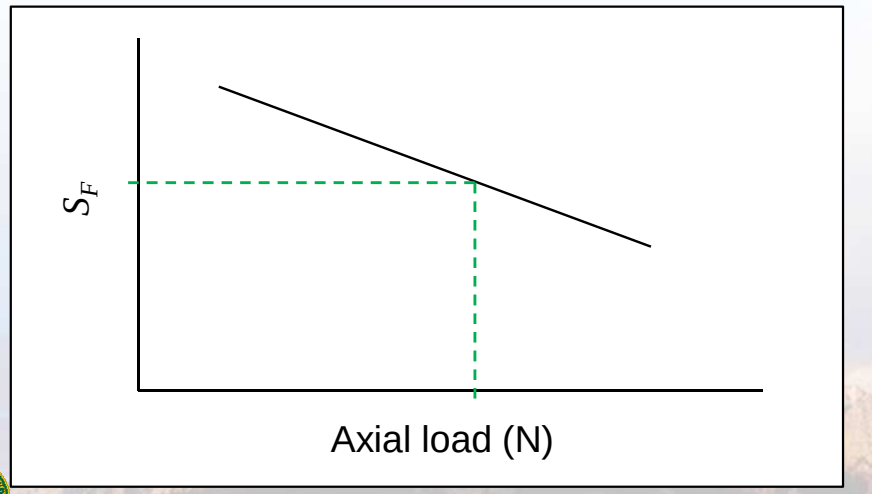
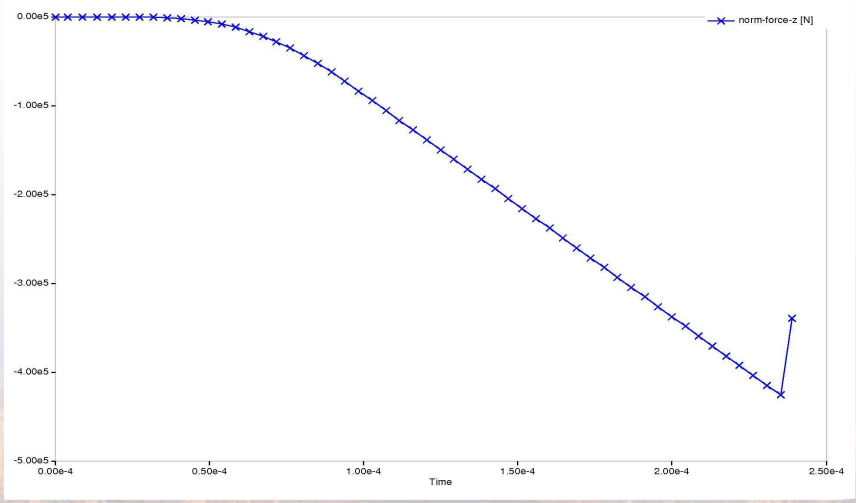
Unconfined Compression Response

- **Triaxial, Unconfined Compression**
- **Test specimen between tapered platens with a range of confining pressure**
- **Loading to failure (loss of bearing capacity) allows identification of limit surface.**
- **For unconfined case, failure related to $S_{F,}$ (fslope)**







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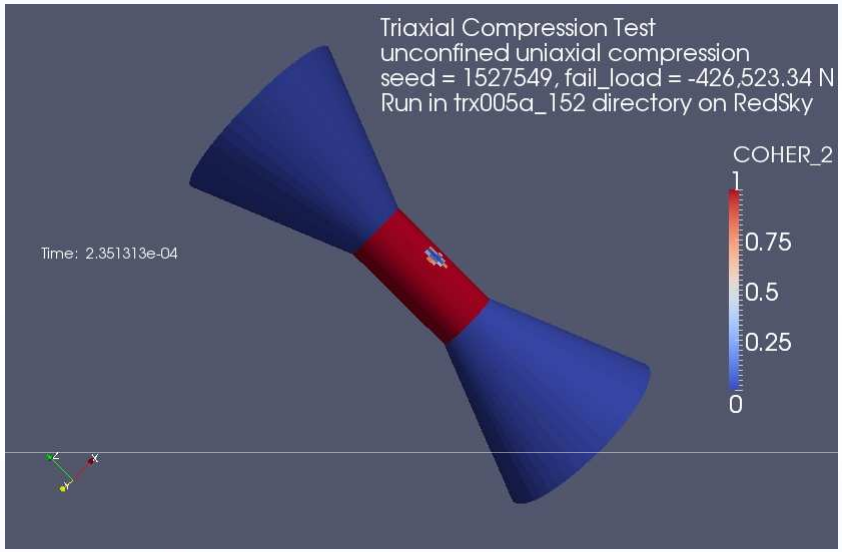

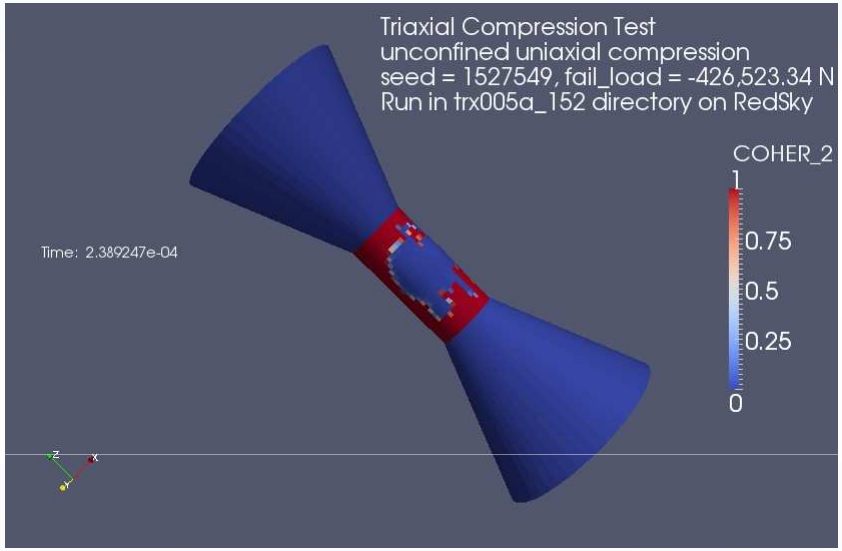

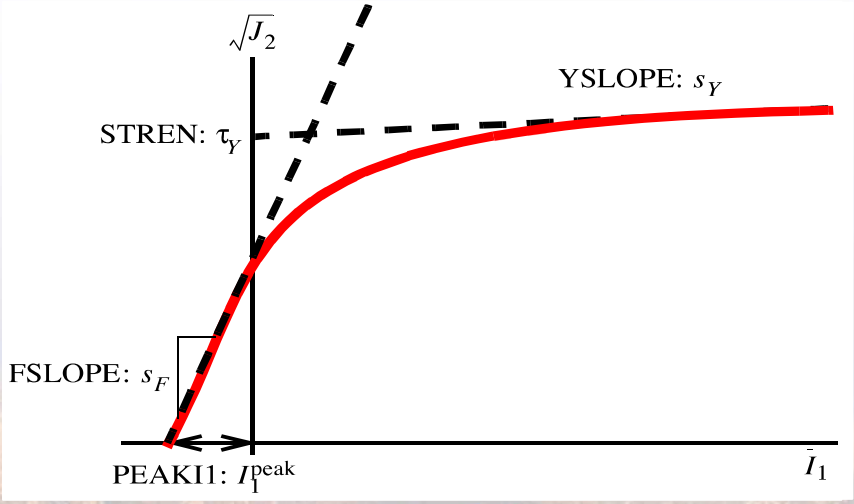

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Concluding Remarks

- Kayenta material model successfully integrated with mechanical EOS and void insertion submodels.
- Capability to manage tensile failure and dilatation
- Spatial variability used to simulate experimental results with failure localization. Provide methodology for calibrating material parameters from experimental data.